CHAOS

Homoclinic Bifurcations, Strange Attractors,
Arnold Diffusion, Fermi Acceleration, Solitons

A Celebration of the 60th Birthday of *Dmitry Turaev*

25-29 September 2023

Nesin Math Village, Izmir, Turkey

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Contents

About

The workshop "Chaos: Homoclinic Bifurcations, Strange Attractors, Arnold Diffusion, Fermi Acceleration, Solitons", a celebration of the 60th birthday of Dmitry Turaev, will be held at the Nesin mathematics village, Şirince, İzmir, Turkey, from 25 to 29 September 2023. The main theme of the workshop is dynamical systems and mathematical physics, and we will celebrate many seminal contributions of Dmitry Turaev to these fields.

Organizing committee

- Jeroen Lamb (Imperial College London, London, UK)
- Deniz Eroglu (Kadir Has University, Istanbul, Turkey)
- Arash M. Rezaeinazhad (Kadir Has University, Istanbul, Turkey)
- Sajjad Bakrani (Kadir Has University, Istanbul, Turkey)
- Zeray Hagos (Kadir Has University, Istanbul, Turkey)
- Asli Can Korkmaz (Nesin Math Village, Turkey)

Programme

The workshop programme (Monday 25/09 to Friday 29/09) is presented in the following table.

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$*$: Boat trip will be arranged based on the weather condition.

The costs for boat trip (lunch included) and Ephesus are respectively 30 and 40 Euros.

Invited Speakers and Corresponding Titles

Speaker	Title of Talk
Invited Talks (40 minutes)	
A. Shilnikov	Bifurcation structure of interval maps with orbits homoclinic to a saddle-focus
O. Pochinka	On the topology of 3-manifolds admitting Morse-Smale diffeomorphisms with fixed points of pairwise different Morse indices
K. Kourliouros	Persistence of Minimal Invariant Sets for Certain Set-Valued Dynamical Systems: A Boundary Map Approach
A.J. Homburg	Iterated function systems of linear expanding and contracting maps on the unit interval
T. Pereira	Chaotic Behavior in Diffusively Coupled Systems
S. van Strien	A survey on some recent developments in 1D dynamics
A. Gonchenko	On discrete Lorenz-like attractors in three-dimensional maps with axial symmetry
V. Rom-Kedar	On some impact-like systems
T. Lazaro	A semiclassical approach for an stochastic saddle-node scaling law
V. Gelfreich	Discrete averaging for nearly integrable symplectic maps
A. Kazakov	On new exampls of pseudohyperbolic attractors
S. Zelik	Spatial and temporal averaging in the theory of inertial manifolds
A. Kirilov	On formation of relic wormholes
K. Safonov	Bifurcations and chaotic attractors in systems with Z^4-symmetry
O. Makarenkov	On the dependence of periodic trajectories of dispersing billiards on parameters
P. Berger	Analytic pseudo-rotations
M. Gonchenko	Reversible perturbations and resonances
E. Nijholt	High-order phase reduction using the parameterization method
D. Li	Symplectic blenders and persistence of homoclinics to saddle-center periodic orbits
N. Stankevich	Discrete and singular Shilnikov attractors in neuron-like models
A. Kostianko	Smoothness of inertial manifolds

Short Talks and Poster Presentations

Speaker	Title of Talk
Short Talks (20 minutes)	
I. Remizov	Dmitry Turaev's contribution to proof of the formula $R(t) = \exp(i(S(t) - I))$ which helps to obtain solutions to Schrödinger equation
M. Barinova	On a structure of non-wandering set of an Ω-stable 3-diffeomorphism possessing a hyperbolic attractor
Z. Hagos	Synchronization transitions caused by time-varying coupling functions
V. Goverse	The quasi-stationary and quasi-ergodic measures for the random logistic map with escape
B. Bassols	Noise-induced chaos: a conditioned random dynamics perspective
K. Czudek	Random walks in quasiperiodic environment
D. Mints	Hidden degeneracies and flat homoclinic tangencies for multidimensional diffeomorphisms
E. Karatetskaia	On new types of Lorenz-like attractors in 3D systems
G. Tenaglia	Topological dynamics for chaotic circle endomorphisms
Posters*	
E. Nozdrinova	On classes of stable isotopic connectivity of gradient-like diffeomorphisms of surfaces
E. Tsaplina	Criterion for the existence of a connected characteristic space of orbits in a gradient-like diffeomorphisms of a surface
A. Dobrolubova	Topological conjugacy of the simplest non-singular three-dimensional flows
D. Shubin	Dynamical systems without fixed points on Seifert fiber spaces

$*$: Poster presentations will be carried out during cake breaks.

Invited Talks

Bifurcation structure of interval maps with orbits homoclinic to a saddle-focus

Carter Hinsley[1], **James Scully**[2], **and Andrey L. Shilnikov**[3], [1,2,3]Georgia State University, Atlanta, USA

We study homoclinic bifurcations in an interval map associated with a saddle-focus of (2, 1)-type in Z2-symmetric systems. Our study of this map reveals the homoclinic structure of the saddle-focus, with a bifurcation unfolding guided by the codimension-two Belyakov bifurcation. We consider three parameters of the map, corresponding to the saddle quantity, splitting parameter, and focal frequency of the smooth saddle-focus in a neighborhood of homoclinic bifurcations. We symbolically encode dynamics of the map in order to find stability windows and locate homoclinic bifurcation sets in a computationally efficient manner. The organization and possible shapes of homoclinic bifurcation curves in the parameter space are examined, taking into account the symmetry and discontinuity of the map. Sufficient conditions for stability and local symbolic constancy of the map are presented. This study furnishes insights into the structure of homoclinic bifurcations of the saddle-focus map, furthering comprehension of low-dimensional chaotic systems.

A survey on some recent developments in 1D dynamics

Sebastian van Strien , Imperial College London, UK

In this talk I will discuss some recent results in the field of 1D dynamics, namely the density of hyperbolic maps within one parameter families of real analytic maps. This result is new, and answers several conjectures that had been open for over 20 years. The proof relies on another recent result which shows that each topological conjugacy class is a real analytic manifolds. I will compare these results to the circle and the higher dimensional case.

On the topology of 3-manifolds admitting Morse-Smale diffeomorphisms with fixed points of pairwise different Morse indices

Olga Pochinka, HSE University, Nizhny Novgorod, Russia

In the present report we consider class G of orientation preserving Morse-Smale diffeomorphisms f, which defined on closed 3-manifold M^3, and whose non-wandering set consist of four fixed points with pairwise different Morse indices. It follows from S.Smale and K. Meyer results that all gradient-like flows with similar properties has Morse energy function with four critical points of pairwise different Morse indices. This implies, that supporting manifold M^3 for these flows admits a Heegaard decomposition of genus 1 and hence it is homeomorphic to a lens space $L_{p,q}$. Despite the simple structure of the non-wandering set in class G there exist diffeomorphisms with wild embedded separatrices. According to V.Grines, F. Laudenbach, O. Pochinka results such diffeomorphisms do not possesses an energy function, and question about topology their supporting manifold is open. According to V. Grines, E. Zhuzhoma and V. Medvedev results M^3 is homeomorphic to a lens space $L_{p,q}$ in case of tame embedding of closures of one-dimensional separatrices of diffeomorphism f from G. Moreover, the wandering set of f contains at least p non-compact heteroclinic curves. In the present paper similar result was received for arbitrary diffeomorphisms of class G. Also we construct diffeomorphisms from G with wild embedding one-dimensional separatrices on every lens space $L_{p,q}$. Such examples were known previously only on the 3-sphere.

Persistence of Minimal Invariant Sets for Certain Set-Valued Dynamical Systems: A Boundary Map Approach

Konstantinos Kourliouros, Imperial College London, UK

We study the problem of persistence of minimal invariant sets with smooth boundary for a certain class of discrete-time set-valued dynamical systems, naturally arising in the context of random dynamical systems with bounded noise. In particular, we introduce a single-valued map, the so-called boundary map, which has the property that a certain class of invariant submanifolds for this map is in one-to-one correspondence with invariant sets for the corresponding set-valued map. We show that minimal invariant sets with smooth boundary persist under small perturbations of the set-valued map, provided that the associated boundary map is normally hyperbolic at the unit normal bundle of the boundary.

This is a joint work with J. S. W. Lamb, M. Rasmussen, W. H. Tey, K. G. Timperi and D. Turaev.

Iterated function systems of linear expanding and contracting maps on the unit interval

Ale Jan Homburg[1,2], [1]KdV Institute for Mathematics, University of Amsterdam and [2]Mathematical Institute,Leiden University

Classical iterated function systems, used to construct fractals, are built from finitely many contracting maps that are iterated randomly. We consider iterated function systems on the interval built from both expanding and contracting maps. We discuss how the dynamics depends on the Lyapunov exponent.

Chaotic Behavior in Diffusively Coupled Systems

Tiago Pereira, University of Sao Paulo, Sao Carlos, Sao Paulo, Brazil

We study emergent oscillatory behavior in networks of diffusively coupled nonlinear ordinary differential equations. Starting from a situation where each isolated node possesses a globally attracting equilibrium point, we give, for an arbitrary network configuration, general conditions for the existence of the diffusive coupling of a homogeneous strength which makes the network dynamics chaotic. The method is based on the theory of local bifurcations we develop for diffusively coupled networks. We, in particular, introduce the class of the so-called versatile network configurations and prove that the Taylor coefficients of the reduction to the center manifold for any versatile network can take any given value.

On discrete Lorenz-like attractors in three-dimensional maps with axial symmetry

Alexandr Gonchenko, National Research University Higher School of Economics, 25/12 Bolshaya Pecherskaya Ulitsa, 603155 Nizhny Novgorod, Russia

We select a class of three-dimensional maps with the axial symmetry $\{x \to -x, y \to -y, z \to z\}$ and the constant Jacobian. We study bifurcations and chaotic dynamics in quadratic 3D maps from this class and show that these maps can possess discrete Lorenz-like attractors of various types. We give a description of bifurcation scenarios leading to such attractors and show examples of their implementation in our maps. We describe also main geometric properties of the discrete Lorenz-like attractors.This work was carried out in the framework of the Russian Science Foundation (project 23-71-30008).

On some Impact-like systems

Vered Rom-Kedar, The Weizmann Institute, Rehovot, Israel

Near integrable Hamiltonian flows with impacts that respect the symmetries of the integrable structure, either globally or locally, provide classes of non-smooth near integrable, or, respectively, near Quasi-Integrable (QI), systems [1-3]. Their extension to smooth systems with large gradients at the domains' boundary, namely to impact-like systems [4], allows, in the near-integrable case, away from grazing tori, despite the non-smooth limit of such systems, to utilize KAM theory [1]. Near grazing tori, the return map of the impact system has a piecewise-smooth rotation with a square-root singularity. Under perturbation, the corresponding tangency-standard-map is derived and is shown to exhibit long transients that are yet to be explained [5].

While the level sets of the integrable impact systems are tori, QI systems also include level sets of genus 2 and higher [3]. Ergodicity on levels sets is proved for some classes of QI Hamiltonian impact systems for a full measure of iso-energy level sets [6] (yet, quantization of such systems suggests that their wavefunctions do not equidistribute in the configuration space in the large energy limit [7]). Return maps for the near-QI systems are shown to be piecewise smooth symplectic maps which are close to families of interval exchange maps. Ongoing projects (e.g. [8]) include the development and study of simplified models for these return maps: regular and singular resonances emerge, as well as transient behavior, leading to conjectures regarding the non-existence of dividing circles in the corresponding singularity band.

References

[1] M. Pnueli and V. Rom-Kedar, SIADS, 17(4), 2018
[2] M. Pnueli and V. Rom-Kedar, Nonlinearity 34 (4), 2021
[3] L. Becker, S. Elliott, B. Firester, S. Gonen Cohen, M. Pnueli and V. Rom-Kedar, 2020/23, MSRI proc.
[4] M. Kloc and V. Rom-Kedar, SIADS 13 (3), 2014.
[5] M. Pnueli and V. Rom-Kedar, SIADS, 21 (3), 2022
[6] K. Fraczek and V. Rom-Kedar, Erg. Theory Dyn. Sys., 67, 2022
[7] O. Yaniv and V. Rom-Kedar, Phys. Rev. E 107, 054221, 2023
[8] A. Zobova and V. Rom-Kedar, 2023, draft.

A semiclassical approach for an stochastic saddle-node scaling law

J. Tomás Lázaro, Universitat Politècnica de Catalunya (BarcelonaTech), Barcelona, Spain

Transients following characteristic scaling laws are common in dynamical systems when they approach a bifurcation or a phase transition. One of them is the well-known saddle-node bifurcation, frequent in many deterministic biological models. It is also known that in this case the time length τ of such 'ghost' transients follows an scaling law of type $\tau \sim |\epsilon - \epsilon_c|^{-1/2}$, where ϵ is the bifurcation parameter and ϵ_c its critical value. This scaling law clearly changes when intrinsic noise affects such models. In this talk we will present a theoretical basis to tackle this new law, performing a WKB asymptotic approximation of the corresponding Master equation and analysing the derived Hamilton-Jacobi like equation for the action. This will be done for two representative examples of population cooperation, the autocatalytic and Hill models.

This is a joint work with Tomás Alarcón (Centre de Recerca Matemàtica CRM), Carlos Peña (Laboratorio Subterráneo de Canfranc LSC), and Josep Sardanyés (CRM).

Discrete averaging for nearly integrable symplectic maps

Vassili Gelfreich, University of Warwick, UK

It is well known that a near-to-identity map can be approximated by a time one flow of an autonomous vector field with a very high accuracy. Usually this vector field is constructed with the help of power series or an auxiliary non-autonomous flow. Recently we invented a method which allows a direct computation of such vector fields. The method is based on interpolation techniques and provides an expression for the vector field in the form of a weighted average of several iterates of the map. In addition to being a useful numerical tool, our method gives a new insight onto some classical theorems.

In this talk we are going to discuss application of the discrete averaging to long time stability of actions in a near-to-integrable symplectic map which leads to a new proof of the classical Nekhoroshev theorem.

The talk is based on a joint work with A.Vieiro.

New examples of pseudohyperbolic attractors

Alexey Kazakov, University Higher School of Economics

In 1998 Turaev and Shilnikov introduce a new class of chaotic attractors – the so-called pseudohyperbolic attractors. The common feature of such attractors is the existence of a continuous splitting of the tangent space in a neighborhood of an attractor into a direct sum of two invariant linear subspaces E1 and E2: the linearized system restricted to E2 is uniformly contracting, whereas in E1 it uniformly expands volumes. Any possible contraction in E1 should be uniformly weaker than any contraction in E2. These guarantee both the positivity of the maximal Lyapunov exponent and the preservation of this property at small perturbations of the system.

To the moment it is known only several examples of pseudohyperbolic attractors in both system with continuous and discrete time. Lorenz and discrete Lorenz attractors are one of the well-known examples of such attractors. The Lorenz attractor is an attractor satisfying conditions of the Afraimovich-Bykov-Shilnikov geometrical model; it contains one saddle equilibrium state with its unstable manifold. The discrete Lorenz attractor is an analogue of the Lorenz attractor which is observed in maps. It has similar "shape" but, unlike the Lorenz attractor, contains orbits of homoclinic and heteroclinic tangencies. Note that only examples of the so-called "flow-like" discrete Lorenz attractors possessing close to zero second Lyapunov exponent are known to the moment. In this talk we will present pseudohyperbolic attractors of two new types: the Lorenz-like attractor containing a pair of saddle equilibria [1] and a "genuinely discrete" Lorenz attractor possessing negative second Lyapunov exponent [2].

References

[1] V. Koryakin, D. Sukharev, A. Kazakov. On Lorenz attractor containing a pair of saddle equilibrium states (in preparation).

[2] A. Kazakov, A. Murillo, A. Vieiro, K. Zaichikov. Numerical study of discrete Lorenz-like attractors (in preparation).

The author is supported by the Laboratory of Dynamical Systems and Applications NRU HSE, the grant of the Ministry of Science and higher education of the RF, ag. № 075-15-2022-1101.

Spatial and temporal averaging in the theory of inertial manifolds

Anna Kostianko[1] **and Sergey Zelik**[2]**,** [1]Imperial College London, [2]University of Surrey, UK

We discus the recent generalizations of the method of spatial averaging for constructing the inertial manifolds for dissipative PDEs. The results will be illustrated on the example of 3D complex Ginzburg-Landau equation with periodic boundary conditions and other physically relevant equations.

On formation of relic wormholes

Alexandr Kirillov, Nizhny Novgorod, Russia

It is shown that in the presence of virtual wormholes, the vacuum is unstable, which leads to a series of phase transitions in the early Universe. Then the standard Kibble scenario predicts the formation of defects such as domain walls. An unusual feature of virtual wormholes is that they generate defects with negative energy density. Such defects have macroscopic dimensions and can support the necks of already real primary wormholes, which gives reason to consider relic wormholes as realistic astrophysical objects.

Symplectic blenders and persistence of homoclinics to saddle-center periodic orbits

Dongchen Li, Imperial College London, UK

A blender is a hyperbolic basic set such that non-transverse intersections with its invariant sets can be unremovable by small perturbations. We show that for any C^r ($r = 4, \ldots, \infty, \omega$) symplectic diffeomorphism of a $2N$-dimensional ($N > 1$) symplectic manifold, symplectic blenders can be obtained by an arbitrarily small symplectic perturbation near any one-dimensional whiskered KAM-torus which has a homoclinic orbit. Using this result, we prove that non-transverse homoclinic intersections between invariant manifolds of a saddle-center periodic point (i.e., it has exactly one pair of complex multipliers on the unit circle) are persistent in the sense that the original map is on the C^r boundary of a C^1 open set in the space of C^r symplectic diffeomorphisms, where maps having such saddle-center homoclinic intersections are dense. These results also hold for Hamiltonian flows in the corresponding settings.

Smoothness of inertial manifolds

Anna Kostianko, Imperial College London, UK

It is well known that in general, we cannot expect more than $C^{1,\varepsilon}$-regularity for inertial manifolds (for some positive, but small ε). Nevertheless, as we will show in the talk, under the natural assumptions, the obstacles to the existence of a C^n-smooth inertial manifold (where $n \in N$ is any given number) can be removed by increasing the dimension and by modifying properly the nonlinearity outside of the global attractor (or even outside the $C^{1,\varepsilon}$-smooth IM of a minimal dimension).

Bifurcations and chaotic attractors in systems with Z^4-symmetry

Klim Safonov, National Research University Higher School of Economics, Russia

In this talk we consider a family $\dot{X} = G_\eta(X)$ of 3-dimensional vector fields, which is invariant with respect to a linear symmetry S generated by the matrix

$$S = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Assume that the family has a symmetric equilibrium state O whose three eigenvalues are equal to 0 at $\eta = 0$. In appropriate coordinates, the system $\dot{X} = G_\eta(X)$ can be written as the following system of differential equations

$$\begin{aligned} \dot{u} &= (-\gamma + i\beta)\, u + a_0\, z u^* + a_1\, u^2 u^* + a_2\, z^2 u + a_3\, (u^*)^3 + O(|u|^4 + |z|^4), \\ \dot{z} &= \mu\, z + b_0\, u^2 + b_0^*\, (u^*)^2 + b_1\, z^3 + b_2\, z u u^* + O(|u|^4 + |z|^4), \end{aligned} \tag{1}$$

where $u = x + iy$ is a complex variable, and the parameters γ, β, μ vanish at the moment of bifurcation. We discuss bifurcations of system (1) and show that if the following conditions hold

$$b_1 < 0, \quad \mathrm{Re}\,(a_0 b_0) < 0, \quad \mathrm{Im}\,(a_0, b_0) \neq 0,$$

then system (1) possesses chaotic pseudohyperbolic attractors for sufficiently small γ, β, μ.

We show that these attractors are of two types. The first type is a symmetric pair of Lorenz attractors and the scenario of creation of these attractors is similar to the scenario in the Shimizu-Morioka system [1]. The second type of attractors is a new type that we call a four-winged Simo attractor. A discrete analogue of such attractors was first observed in [2]. For the four-winged Simo attractor, we construct a geometric model similar to the geometric model of the Lorenz attractor and provide a criterion for the birth of such attractors as a result of a certain heteroclinic bifurcation.

At last, we discuss the application of our result to the study of a fixed point with multipliers $(i, -i, -1)$ of a three-dimensional map.

Acknowledgments. This work is supported by Laboratory of Dynamical Systems and Applications NRU HSE, grant of the Ministry of science and higher education of the RF, ag. No. 075-15-2022-1101.

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[1] A. Shil'nikov. "On bifurcations of the Lorenz attractor in the Shimizu-Morioka model." Physica D: Nonlinear Phenomena, 1993, V.62, p. 338-346.

[2] S. Gonchenko, I. Ovsyannikov, C. Simo, D. Turaev. "Three-dimensional Hénon-like maps and wild Lorenz-like attractors." International Journal of Bifurcation and Chaos, 2005, V. 15(11), p. 3493-3508.

Discrete and singular Shilnikov attractors in neuron-like models

Nataliya Stankevich, HSE University, Russia

Neuron models are one of the most important objects in neurophysiology, neurodynamics, machine learning, nonlinear dynamics, etc. These models are used as a basis for modeling various real-life applications of wide rang: from investigation of isolated (single) cell till modeling of brain functioning. There are a large number of models with their own specifics. In this work we consider two types of models: with discrete and continuous time which can demonstrate formation of discrete and singular Shilnikov attractors.

On the dependence of periodic trajectories of dispersing billiards on parameters

Oleg Makarenkov, Nizhny Novgorod, Russia

We estimate the response of an arbitrary periodic trajectory of a planar dispersing billiard to the variation of the geometry of the billiard (such as e.g. displacement of a disc) under assumption that one of the segments of the periodic trajectory collides with a disc almost tangentially. The goal is to design such a perturbation of the billiard which ensures the existence of a periodic trajectory that passes by one of the discs just tangentially. The motivation comes from [Turaev, Rom-Kedar, Nonlinearity 11 (1998)] where the authors discovered that the presence of a tangential periodic orbit leads to the occurrence of an island of stability in the corresponding regularized (smoothen) Hamiltonian flow, thus making the flow non-ergodic. Some of the key computations in the present work are due to Vaughn Osterman (currently PhD student at the University of Maryland).

Analytic pseudo-rotations

Pierre Berger, Université Sorbonne, France

We construct analytic symplectomorphisms of the cylinder or the sphere with zero or exactly two periodic points and which are not conjugated to a rotation. In the case of the cylinder, we show that these symplectomorphisms can be chosen ergodic or to the contrary with local emergence of maximal order. In particular, this disproves a conjecture of Birkhoff (1941) and solve a problem of Herman (1998). One aspect of the proof provides a new approximation theorem, it enables in particular to implement the Anosov-Katok scheme in new analytic settings

Reversible perturbations and resonances

Marina Gonchenko, Universitat Politècnica de Catalunya, Spain

We consider area-preserving reversible maps and study smooth perturbations that keep the reversibility of the initial maps but destroy their conservativity. To construct these perturbations, we use two methods, a new method based on reversible properties of maps written in the so-called cross-form, and the classical Quispel-Roberts method based on a variation of involutions of the initial map. We study symmetry breaking bifurcations of symmetric periodic orbits in reversible families. In particular, we study how reversible non-conservative perturbations affect to the structure of the p:q resonance with odd $q \geq 3$, i.e. bifurcations of fixed points with eigenvalues $e^{\pm ip\pi/q}$. We prove the birth of garlands consisting of four q-periodic orbits, and describe the corresponding bifurcation diagrams for one- and two-parameter families.

This is a joint work with S. Gonchenko, K. Safonov, A. Kazakov, E. Samylina and A. Shykhmamedov.

High-order phase reduction using the parameterization method

Eddie Nijholt, Imperial College London, UK

Networks of coupled oscillators are some of the best models we have for understanding complex systems. Assuming weak coupling, many questions deal with the behavior on or near the corresponding invariant torus that continues from the uncoupled situation. A major hurdle in this so-called phase-reduction is the difficulty in obtaining expressions for higher-order terms in the small parameter. We present a novel way of performing high-order phase reduction, using the parameterization method. Our techniques only require the uncoupled oscillators to be hyperbolic, and directly allow us to obtain phase-dynamics that are in normal form (i.e. with only resonant terms up to arbitrary finite order). We end with an example that shows remote synchronisation in a network of three coupled Stuart-Landau oscillators. This is joint work with Sören von der Gracht and Bob Rink.

Short Talks

Dmitry Turaev's contribution to proof of the formula R(t)=exp(i(S(t)-I)) which helps to obtain solutions to Schrödinger equation

Ivan Remizov, HSE University (Nizhny Novgorod, Russia)

In the talk I will highlight the short, but bright contribution of Professor Dmitry Turaev to the proof of the main theorem in the following paper:

Ivan D. Remizov. Quasi-Feynman formulas – a method of obtaining the evolution operator for the Schrödinger equation/Journal of Functional Analysis, 270:12 (2016)

On this concrete example I will try to demonstrate my impression of how his beautiful mind works, and what a great colleague he is.

On a structure of non-wandering set of an Ω-stable 3-diffeomorphism possessing a hyperbolic attractor

Marina Barinova, HSE University, Russia

The results were obtained in collaboration with Olga Pochinka and Evgeniy Yakovlev.

Let M^n be a smooth closed connected n-manifold with a Riemannian metric d and $f : M^n \to M^n$ be a diffeomorphism and an invariant compact set $\Lambda \subset M^n$ is hyperbolic. The hyperbolic structure of Λ implies the existence of stable and unstable manifolds W^s_x, W^u_x respectively for any point $x \in \Lambda$:

$$W^s_x = \{y \in M^n : \lim_{j\to+\infty} d(f^j(x), f^j(y)) = 0\},$$

$$W^u_x = \{y \in M^n : \lim_{j\to+\infty} d(f^{-j}(x), f^{-j}(y)) = 0\},$$

which are smooth injective immersions of the stable and unstable subbundles E^s_x and E^u_x into M^n. Moreover, W^s_x, W^u_x are tangent to E^s_x and E^u_x at x respectively. For $r > 0$ we will denote by $W^s_{x,r}$, $W^u_{x,r}$ the immersions of discs on the subbundles E^s_x, E^u_x of the radius r.

If the chain-recurrent set R_f of f is hyperbolic then f is an Ω-stable. In this case the chain-recurrent set is a finite union of pairwise disjoint sets, called *basic sets*

$$R_f = \Lambda_1 \sqcup \cdots \sqcup \Lambda_m,$$

each of which is compact, invariant and topologically transitive. A basic set Λ_i of an Ω-stable diffeomorphism $f : M^n \to M^n$ is called *trivial* if it coincides with a periodic orbit and *non-trivial* in the opposite case.

A non-trivial basic set Λ_i is called *orientable* if for any point $x \in \Lambda_i$ and any fixed numbers $\alpha > 0$, $\beta > 0$ the intersection index $W^u_{x,\alpha} \cap W^s_{x,\beta}$ is the same at all intersection points ($+1$ or -1) [1]. Otherwise, the basic set is called *non-orientable*.

A basic set Λ_i is called an *attractor* if there exists a compact neighborhood U_{Λ_i} (*a trapping neighborhood*) of Λ_i such that $f(U_{\Lambda_i}) \subset \operatorname{int} U_{\Lambda_i}$ and $\Lambda_i = \bigcap\limits_{i=0}^{\infty} f^i(U_{\Lambda_i})$. Due to [2], a non-trivial attractor Λ_i of f is said to be *expanding* if $\dim \Lambda_i = \dim W^u_x$, $x \in \Lambda_i$.

The main result of this paper is following.

Theorem 1 *Let $f : M^3 \to M^3$ be an Ω-stable diffeomorphism whose basic sets are trivial except attractors. Then every non-trivial attractor is either one-dimensional non-orientable or two-dimensional expanding.*

Notice, that the attractors of both types described in the Theorem 1 are realized. In particular, the Figure 1 shows a phase portrait of a structurally stable diffeomorphism of a 3-sphere, whose non-wandering set consists of a one-dimensional non-orientable Plykin attractor, four saddle points with a two-dimensional unstable manifold and two sources. The DA-diffeomorphism of 3-torus on Figure 2 is an example of a combination of an orientable two-dimensional expanding attractor with a source in the non-wandering set of a structurally stable diffeomorphism.

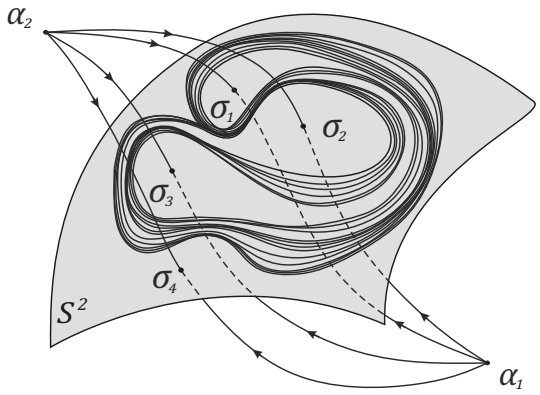

Figure 1: Ω-stable diffeomorphism $f : \mathbb{S}^3 \to \mathbb{S}^3$ with the unique non-trivial basic set which is Plykin attractor

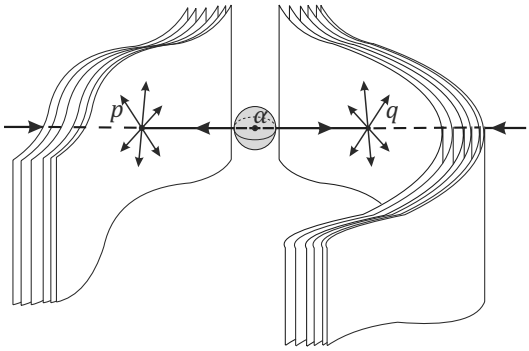

Figure 2: DA-map on $\mathbb{T}^3$

Thanks. The author is partially supported by Laboratory of Dynamical Systems and Applications NRU HSE, grant of the Ministry of science and higher education of the RF, ag. № 075-15-2022-1101.

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Random walks in quasiperiodic environment

Klaudiusz Czudek, Imperial College London, UK

The talk will be devoted to random walks in random environment. A useful tool in the study of these random walks is the environment viewed by the particle process. I will present what is known about invariant measures of that process in the context of so called quasiperiodic environments.

Synchronization transitions caused by time-varying coupling functions

Zeray Hagos Gebrezabher, Kadir Has University, Istanbul, Turkey

Interacting dynamical systems are widespread in nature. The influence that one such system exerts on another is described by a coupling function. Although much effort has been devoted to the analysis of coupling functions, the influence of time-variability on the associated dynamics remains largely unexplored. Motivated especially by coupling functions in biology, including the cardiorespiratory and neural delta-alpha coupling functions, this talk offers a contribution to the understanding of effects due to time-varying interactions. Through both numerics and mathematically rigorous theoretical consideration, we show that for time-variable coupling functions with time-independent net coupling strength, transitions into and out of phase- synchronization can occur, even though the frozen coupling functions determine phase-synchronization solely by virtue of their net coupling strength. Thus the information about interactions provided by the shape of coupling functions plays a greater role in determining behaviour when these coupling functions are time-variable.

This is a joint work with Tomislav Stankovski, Julian Newman, Tiago Pereira, Peter V. E. McClintock, and Aneta Stefanovska.

Noise-induced chaos: a conditioned random dynamics perspective

Bernat Bassols Cornudella, Imperial College London, UK

We consider transitions to chaos in random dynamical systems induced by an increase of noise amplitude. We show how the emergence of chaos (indicated by a positive Lyapunov exponent) in a logistic map with bounded additive noise can be analysed in the framework of conditioned random dynamics through expected escape times and conditioned Lyapunov exponents for a compartmental model representing the competition between contracting and expanding behaviour. We find that the noise-induced transition to chaos is caused by a rapid decay of the expected escape time from the contracting compartment, while all other order parameters remain approximately constant.

The quasi-stationary and quasi-ergodic measures for the random logistic map with escape

Vincent Goverse, Imperial College London, UK

In this talk we consider the random logistic map $X_{n+1} = \omega_n X_n(1 - X_n)$ absorbed at $\mathbb{R} \setminus [0, 1]$, where ω_n is an i.i.d sequence of random variables uniformly distributed in $[a, b]$, for $1 \leq a < 4$ and $b > 4$.

We apply novel techniques to find the quasi-stationary measure μ. As the transition kernel $\mathcal{P}$ admits an eigenfunction $0 \leq \eta \in L^1(M, \mu)$. We find conditions on the transition densities of $\mathcal{P}$ with respect to μ which ensure that $\eta(x)\mu(\dot{x})$ is a quasi-ergodic measure for X_n and that the Yaglom limit converges to the quasi-stationary measure μ-almost surely.

This is a joint work with: M.M. Castro, J.S.W. Lamb, M. Rasmussen.

New types of Lorenz-like attractors in 3D systems

Efrosiniia Karatetskaia, University Higher Shool of Economics

We describe new types of Lorenz-like attractors for three-dimensional flows and maps with symmetries. We give an example of a three-dimensional system of differential equations which is centrally-symmetric and mirror-symmetric. We show that the system has a Lorenz-like attractor, which contains three saddle equilibrium states and consists of two mirror-symmetric components that are adjacent at the symmetry plane. We also found a discrete-time analog of this "conjoined twins" attractor in a cubic three-dimensional Hénon map with central symmetry. We show numerically that both attractors are pseudohyperbolic, which guarantees that each orbit of the attractor has a positive maximal Lyapunov exponent and this property is preserved under small perturbations. We also describe bifurcation scenarios for the emergence of the attractors in one-parameter families of three-dimensional flows and maps possessing the symmetries. This is a joint work with S. Gonchenko, A. Kazakov, and V. Kruglov.

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Hidden degeneracies and flat homoclinic tangencies for multidimensional diffeomorphisms

Dmitrii Mints, Imperial College London, UK

Our research is aimed at studying the dynamics of smooth multidimensional maps (local diffeomorphisms) from Newhouse domains, that is, open regions in the space of maps where systems with homoclinic tangencies are dense. We prove that maps with flat homoclinic tangencies of corank-1 (invariant manifolds forming tangency of corank-1 have a unique common tangent vector) and maps with arbitrarily degenerate periodic orbits are dense in the Newhouse domains in the space of smooth k-dimensional maps, where $k \geq 2$. We also show that in the space of smooth k-dimensional maps, where $k \geq 4$, in any neighborhood of a map such that it has bi-focus saddle periodic orbit whose invariant manifolds are tangent, there exist open domains in which systems with flat homoclinic tangencies of corank-2 are dense (invariant manifolds forming tangency of corank-2 have a pair of linearly independent common tangent vectors).

Topological dynamics for chaotic circle endomorphisms

Giuseppe Tenaglia, Imperial College London, UK

In this talk we introduce a class of random circle endomorphism displaying positive lyapunov exponent. For such systems we propose a notion of random horseshoe and prove abundance of horseshoe like dynamics.

Posters

On classes of stable isotopic connectivity of gradient-like diffeomorphisms of surfaces

Elena Nozdrinova, HSE University, Russia

In 1976 S. Newhouse, J. Palis and F. Takens introduced a stable arc joining two structurally stable systems on a manifold. Later in 1983 they proved that all points of a regular stable arc are structurally stable diffeomorphisms except for a finite number of bifurcation diffeomorphisms which have no cycles, no heteroclinic tangencies and which have a unique nonhyperbolic periodic orbit, this orbit being the orbit of a noncritical saddle-node or a flip which unfolds generically on the arc. There are examples of Morse – Smale diffeomorphisms on manifolds of any dimension which cannot be joined by a stable arc. There naturally arises the problem of finding an invariant defining the equivalence classes of Morse – Smale diffeomorphisms with respect to connectedness by a stable arc. In the present report we present the classification results for Morse – Smale diffeomorphisms with respect to stable isotopic connectedness and obstructions to existence of stable arcs including the authors' recent results.

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Criterion for the existence of a connected characteristic space of orbits in a gradient-like diffeomorphisms of a surface

Ekaterina Tsaplina, Laboratory of Dynamical Systems and Applications, NRU HSE, Russia

The classical approach to the study of dynamical systems consists in representing the dynamics of the system in the form of a "source-drain", that is, in the allocation of a dual pair of attractor-repeller, which are attractive and repulsive sets for all other trajectories of the system. If it is possible to choose a dual pair of attractor-repeller so that the space of orbits in their complement (the characteristic space of orbits) is connected, then this creates prerequisites for finding complete topological invariants of the dynamical system. In this way, in particular, a number of classification results for Morse-Smale systems were obtained. For instance, the complete topological classification of Morse-Smale 3-diffeomorphisms is essentially based on the presence of a connected characteristic space of orbits associated with the choice of a one-dimensional dual attractor-repeller pair. For Morse-Smale diffeomorphisms with heteroclinic points on surfaces, examples are known for which all characteristic spaces of orbits are not connected. The poster considers the question of the existence of a connected characteristic space of orbits for gradient-like (without heteroclinic points) diffeomorphisms on surfaces. On an orientable surface of any kind, an orientation-changing diffeomorphism is constructed that does not have a connected characteristic space. A gradient-like diffeomorphism without a connected characteristic space is also constructed on an undirectable surface of any kind. A criterion for the existence of a connected characteristic orbit is also established.

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Topological conjugacy of the simplest non-singular three-dimensional flows

Alisa Dobrolubova, Laboratory of Dynamical Systems and Applications NRU HSE, Russia

The simplest non-singular flows on closed orientable 3-manifolds are studied. We establish that each class of topological equivalence of the simplest non-singular flow on a lens consists of an infinite set of topological conjugacy classes. We obtain necessary and sufficient conditions for the topological conjugacy of the flows under consideration.

Thanks: The author is supported by Laboratory of Dynamical Systems and Applications NRU HSE, grant of the Ministry of science and higher education of the RF, ag. № 075-15-2022-1101.

Dynamical systems without fixed points on Seifert fiber spaces

Danila Shubin, Laboratory of Dynamical Systems and Applications NRU HSE, Russia

It is well known that topology and dynamical systems are interlinked. Various topological methods are used in the theory of dynamical systems. One of the basic questions for the theory is: Can a dynamical system with certain properties be defined on a certain topological space? Our talk is dedicated to the class of dynamical systems with continuous time, and whose non-wandering set consists of three hyperbolic periodic orbits (one is attracting, one is repelling, and one is saddle). We show, that if the saddle orbit is twisted, then the ambient manifold is either lens space or Seifert fiber space with three singular fibers.

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